CAL 282

Probability & Statistics

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9th Week

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Introduction

- **Hypothesis testing** is a decision-making process for evaluating claims about a population.
- In hypothesis testing, the researcher must
  - define the population under study,
  - state the particular hypotheses that will be investigated,
  - give the significance level,
  - select a sample from the population,
  - collect the data,
  - perform the calculations required for the statistical test, and
  - reach a conclusion
- Hypotheses concerning parameters such as means and proportions can be investigated. There are two specific statistical tests used for hypotheses concerning means:
  - z test,
  - t test

Introduction

- Three methods used to test hypotheses are
  - Traditional method,
  - P-value method
  - Confidence interval method

Steps in Hypothesis Testing—Traditional Method

- **Every hypothesis-testing situation begins with the statement of a hypothesis**.
- A statistical hypothesis is a conjecture about a population parameter. This conjecture may or may not be true.
- There are two types of statistical hypotheses for each situation: the null hypothesis and the alternative hypothesis.
- The **null hypothesis**, symbolized by $\boldsymbol{H_0}$, is a statistical hypothesis that states that **there is no difference between a parameter and a specific value**, or **that there is no difference between two parameters**.
- The **alternative hypothesis**, symbolized by $\boldsymbol{H_1}$, is a statistical hypothesis that states the **existence of a difference between a parameter and a specific value**, or states that **there is a difference between two parameters**.

Steps in Hypothesis Testing—Traditional Method

- **Two tailed test**
- **Right tailed test**
- **Left tailed test**
- **Example 1:** A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate increase, decrease, or remain unchanged after a patient takes the medication?
- Since the researcher knows that the mean pulse rate for the population under study is 82 beats per minute, the hypotheses for this situation are

$$H_0: \mu = 82 \text{ ve } H_1: \mu \neq 82$$

- The null hypothesis specifies that the mean will remain unchanged, and the alternative hypothesis states that it will be different.

Steps in Hypothesis Testing—Traditional Method

- **Example 2:** A chemist invents an additive to increase the life of an automobile battery. If the mean lifetime of the automobile battery without the additive is 36 months, then her hypotheses are

$$H_0: \mu = 36 \text{ ve } H_1: \mu > 36$$

- In this situation, the chemist is interested only in increasing the lifetime of the batteries, so her alternative hypothesis is that the mean is greater than 36 months. The null hypothesis is that the mean is equal to 36 months.
- **Example 3:** A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is $78, her hypotheses about heating costs with the use of insulation are

$$H_0: \mu = \$78 \text{ ve } H_1: \mu < \$78$$

- This test is a left-tailed test, since the contractor is interested only in lowering heating costs.

Steps in Hypothesis Testing—Traditional Method

- **The null hypothesis is always stated using the equals sign**. This is done because in most professional journals, and when we test the null hypothesis, **the assumption is that the mean, proportion, or standard deviation is equal to a given specific value**.
- When a researcher conducts a study, he or she is generally looking for evidence to support a claim. Therefore, the claim should be stated as the alternative hypothesis, i.e., using $<$ or $>$ or $\neq$. Because of this, the alternative hypothesis is sometimes called the **research hypothesis**.
- A claim, though, can be stated as either the null hypothesis or the alternative hypothesis; **however, the statistical evidence can only support the claim if it is the alternative hypothesis**.
- **Statistical evidence can be used to reject the claim if the claim is the null hypothesis**. These facts are important when you are stating the conclusion of a statistical study.
- After stating the hypothesis, the researcher designs the study. The researcher selects the **correct statistical test**, chooses an **appropriate level of significance**, and formulates a plan for conducting the study.

Steps in Hypothesis Testing—Traditional Method

- **A statistical test** uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.
- The numerical value obtained from a statistical test is called the **test value**.
- In the hypothesis-testing situation, there are four possible outcomes.

	H_0 true	H_0 false
Reject H_0	**Error** Type I	Correct decision
Do not reject H_0	Correct decision	**Error** Type II

Steps in Hypothesis Testing—Traditional Method

- **A type I error** occurs if you reject the null hypothesis when it is true.
- **A type II error** occurs if you do not reject the null hypothesis when it is false.
- The **hypothesis-testing situation** can be likened to a jury trial. In a jury trial, there are four possible outcomes. The defendant is either guilty or innocent, and he or she will be convicted or acquitted.

H_0: The defendant is innocent, H_1: The defendant is not innocent (i.e., guilty)

- The evidence is presented in court by the prosecutor, and based on this evidence, the jury decides the verdict, innocent or guilty.
- If the defendant is convicted but he or she did not commit the crime, then a **type I error** has been committed.
- If the defendant is convicted and he or she has committed the crime, then a correct decision has been made.
- If the defendant is acquitted and he or she did not commit the crime, a correct decision has been made by the jury.
- If the defendant is acquitted and he or she did commit the crime, then a **type II error** has been made

Steps in Hypothesis Testing—Traditional Method

- The decision of the jury does not prove that the defendant did or did not commit the crime. The decision is based on the evidence presented.
- If the evidence is strong enough, the defendant will be convicted in most cases.
- If the evidence is weak, the defendant will be acquitted in most cases. Nothing is proved absolutely. Likewise, the decision to reject or not reject the null hypothesis does not prove anything.
- **The only way to prove anything statistically is to use the entire population, which, in most cases, is not possible**. The decision, then, is made on the basis of probabilities. That is, when there is a large difference between the mean obtained from the sample and the hypothesized mean, the null hypothesis is probably not true. The question is, How large a difference is necessary to reject the null hypothesis? Here is where the level of significance is used.
- **The level of significance** is the maximum probability of committing a type I error. This probability is symbolized by **α (Greek letter alpha)**. That is, **P(type I error)=α**.

Steps in Hypothesis Testing—Traditional Method

- The probability of a type II error is symbolized by β, the Greek letter beta. That is, P(type II error)= β. In most hypothesis-testing situations, b cannot be easily computed; however, α and β are related in that decreasing one increases the other.
- Statisticians generally agree on using three arbitrary significance levels: the **0.10**, **0.05**, and **0.01** levels.
- If the null hypothesis is rejected, the probability of a type I error will be 10%, 5%, or 1%, depending on which level of significance is used.
- When α=0.10, there is a 10% chance of rejecting a true null hypothesis; when α=0.05, there is a 5% chance of rejecting a true null hypothesis; and when α=0.01, there is a 1% chance of rejecting a true null hypothesis.
- In a hypothesis-testing situation, the researcher decides what level of significance to use.
- It can be any level, depending on the seriousness of the type I error.

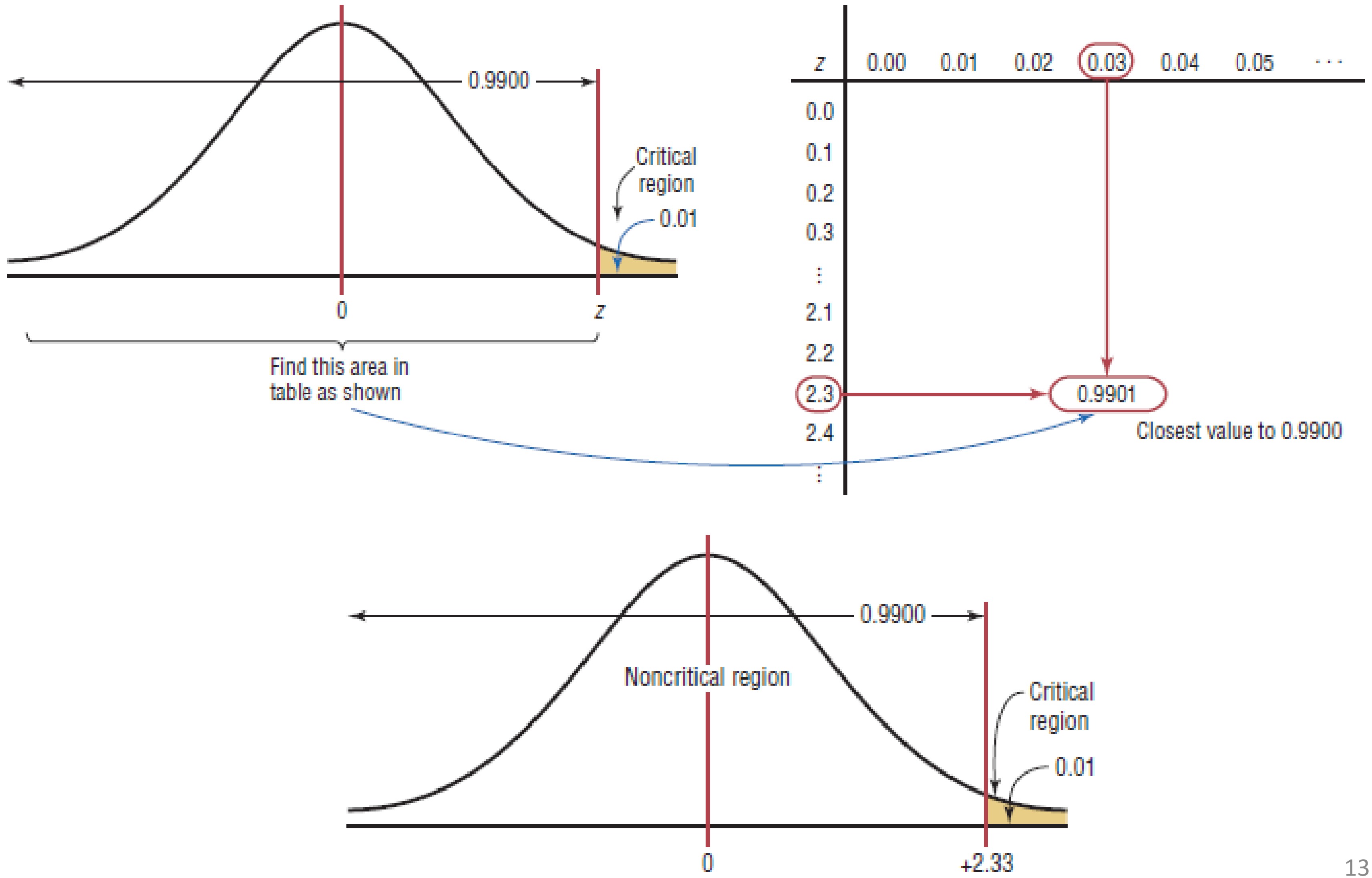
Steps in Hypothesis Testing—Traditional Method

- After a significance level is chosen, a critical value is selected from a table for the appropriate test.
- If a z test is used, for example, the z table is consulted to find the critical value. The critical value determines the critical and noncritical regions.
- The critical value separates the critical region from the noncritical region. The symbol for critical value is C.V.
- **The critical or rejection region** is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.
- The noncritical or nonrejection region is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.
- A **one-tailed test** indicates that the null hypothesis should be rejected **when the test value is in the critical region** on one side of the mean.
- A one-tailed test is either a right tailed test or left-tailed test, depending on the direction of the inequality of the alternative hypothesis.
- In a two-tailed test, the null hypothesis should be rejected when the test value is in either of the two critical regions.

Steps in Hypothesis Testing—Traditional Method

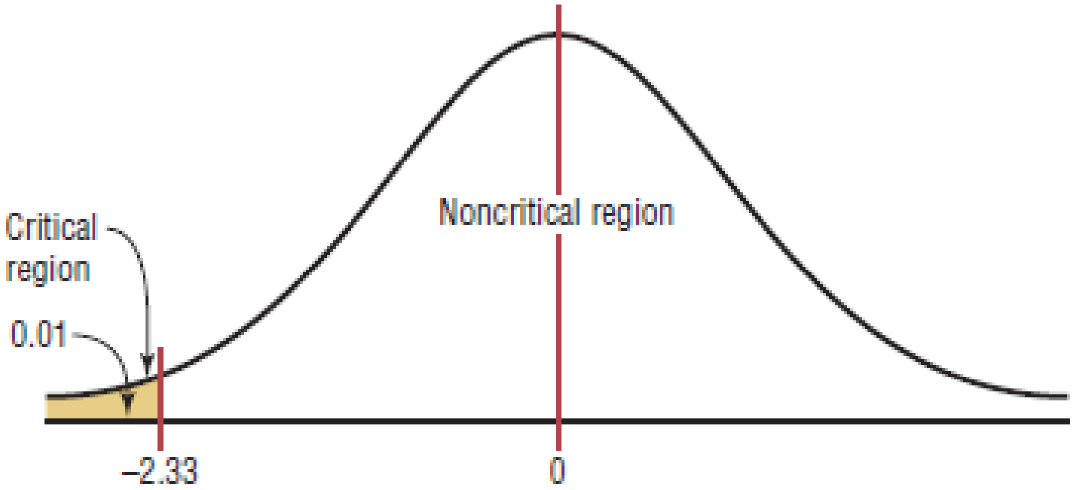
- **Örnek 2:** α=0.01

Finding the Critical Value for α = 0.01 (Right-Tailed Test)



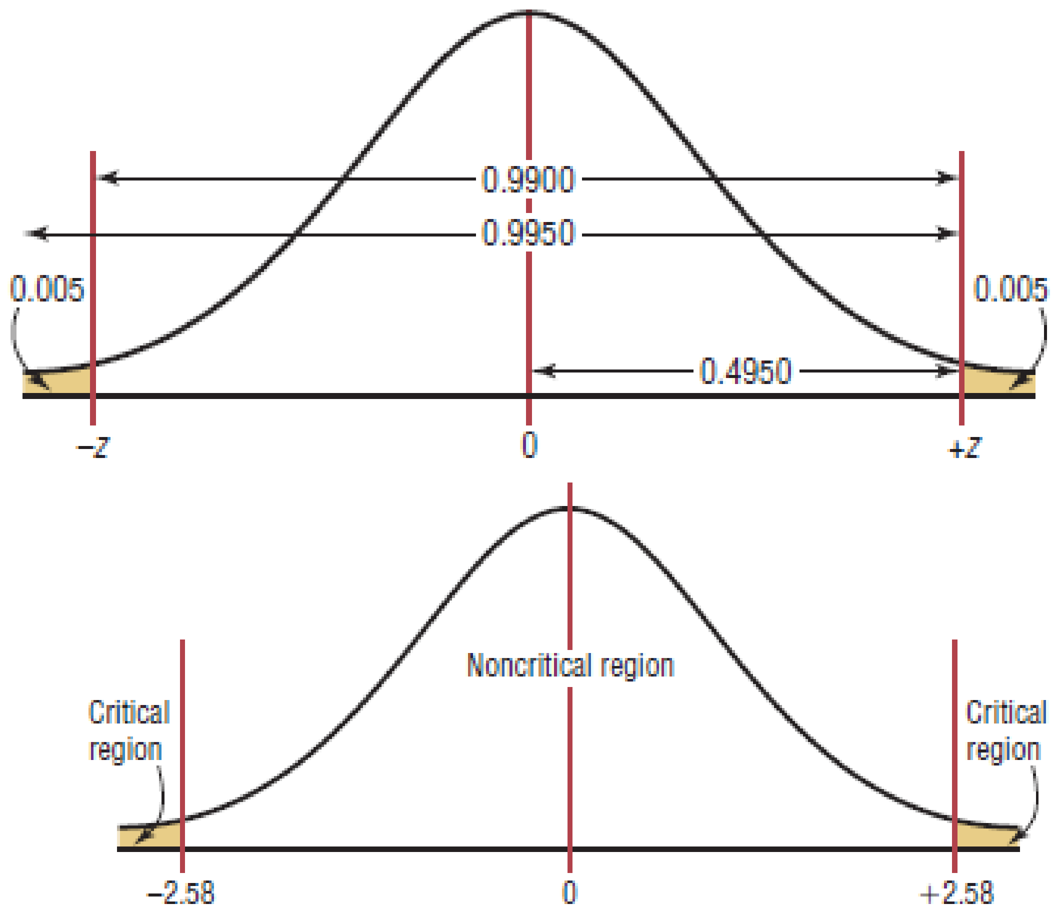
Steps in Hypothesis Testing—Traditional Method

- **Örnek 3:** α=0.01



Steps in Hypothesis Testing—Traditional Method

- **Örnek 1:** α=0.01
- For a two-tailed test, then, the critical region must be split into two equal parts. If α=0.01, then one-half of the area, or 0.005, must be to the right of the mean and one half must be to the left of the mean.



Steps in Hypothesis Testing—Traditional Method

- **Procedure table (Finding the critical values for specific α values):**

Step 1 Draw the figure and indicate the appropriate area.

a. If the test is left-tailed, the critical region, with an area equal to α, will be on the left side of the mean.

b. If the test is right-tailed, the critical region, with an area equal to α, will be on the right side of the mean.

c. If the test is two-tailed, α must be divided by 2; one-half of the area will be to the right of the mean, and one-half will be to the left of the mean.

Step 2 *a.* For a left-tailed test, use the z value that corresponds to the area equivalent to α in Table E.

b. For a right-tailed test, use the z value that corresponds to the area equivalent to $1 - \alpha$.

c. For a two-tailed test, use the z value that corresponds to $\alpha/2$ for the left value. It will be negative. For the right value, use the z value that corresponds to the area equivalent to $1 - \alpha/2$. It will be positive.

Steps in Hypothesis Testing—Traditional Method

- **Hypothesis testing:**
- State the hypotheses. Be sure to state both the null and the alternative hypotheses.
- Design the study. This step includes selecting the correct statistical test, choosing a level of significance, and formulating a plan to carry out the study. The plan should include information such as the definition of the population, the way the sample will be selected, and the methods that will be used to collect the data.
- Conduct the study and collect the data.
- Evaluate the data. The data should be tabulated in this step, and the statistical test should be conducted. Finally, decide whether to reject or not reject the null hypothesis.
- Summarize the results.

Steps in Hypothesis Testing—Traditional Method

- **Hypothesis testing (Solving Hypothesis-Testing Problems (Traditional Method))**
- **Step 1** State the hypotheses and identify the claim.
- **Step 2** Find the critical value(s) from the appropriate.
- **Step 3** Compute the test value.
- **Step 4** Make the decision to reject or not reject the null hypothesis.
- **Step 5** Summarize the results.

z Test For a Mean

- **z test** is used when **σ is known**, and the **t test** is used when **σ is unknown**.
- Many hypotheses are tested using a statistical test based on the following general formula:

$$test\ value = \frac{(observed\ value) - (expected\ value)}{standard\ error}$$

- The observed value is the statistic (such as the sample mean) that is computed from the sample data.
- The expected value is the parameter (such as the population mean) that you would expect to obtain if the null hypothesis were true—in other words, the hypothesized value.
- The denominator is the standard error of the statistic being tested (in this case, the standard error of the mean).

z Test For a Mean

- **z test**

The **z test** is a statistical test for the mean of a population. It can be used when $n \geq 30$, or when the population is normally distributed and σ is known.

The formula for the z test is

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

where

$\overline{X}$ = sample mean
μ = hypothesized population mean
σ = population standard deviation
n = sample size

- For the z test, the observed value is the value of the sample mean. The expected value is the value of the population mean, assuming that the null hypothesis is true. The denominator $\sigma/\sqrt{n}$ is the standard error of the mean.

z Test For a Mean

- **z test**
- There are five steps for solving hypothesis-testing problems:
- **Step 1** State the hypotheses and identify the claim.
- **Step 2** Find the critical value(s).
- **Step 3** Compute the test value.
- **Step 4** Make the decision to reject or not reject the null hypothesis.
- **Step 5** Summarize the results.

z Test For a Mean

- **z test**
- **Example:** A researcher wishes to see if the mean number of days that a basic, low-price, small automobile sits on a dealer's lot is 29. A sample of 30 automobile dealers has a mean of 30.1 days for basic, low-price, small automobiles. At α=0.05, test the claim that the mean time is greater than 29 days. The standard deviation of the population is 3.8 days.
- **Solution:**

Step 1 State the hypotheses and identify the claim.

H_0: $\mu = 29$ and H_1: $\mu > 29$ (claim)

Step 2 Find the critical value. Since $\alpha = 0.05$ and the test is a right-tailed test, the critical value is $z = +1.65$.

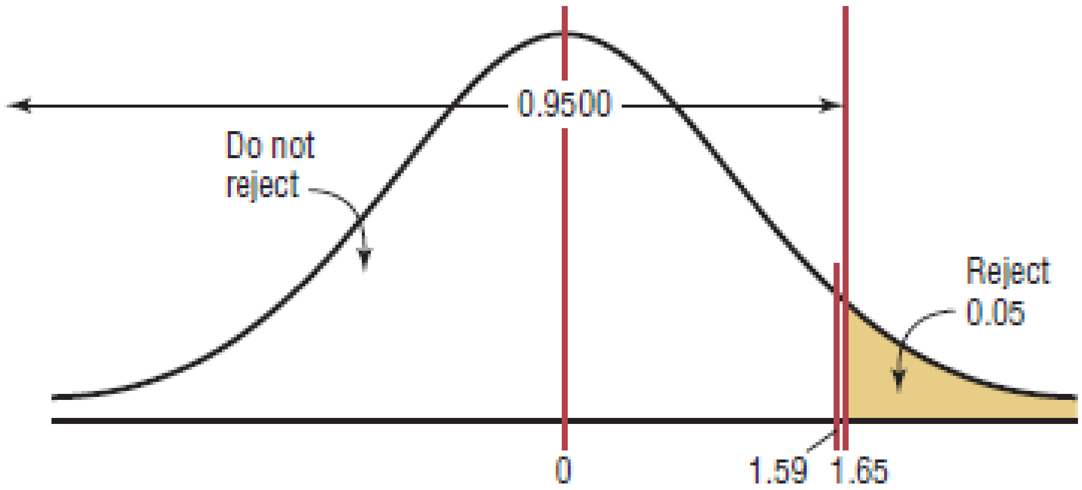
Step 3 Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{30.1 - 29}{3.8/\sqrt{30}} = 1.59$$

Step 4 Make the decision. Since the test value, +1.59, is less than the critical value, +1.65, and is not in the critical region, the decision is to not reject the null hypothesis.

z Test For a Mean

- **z test**
- **Solution:**

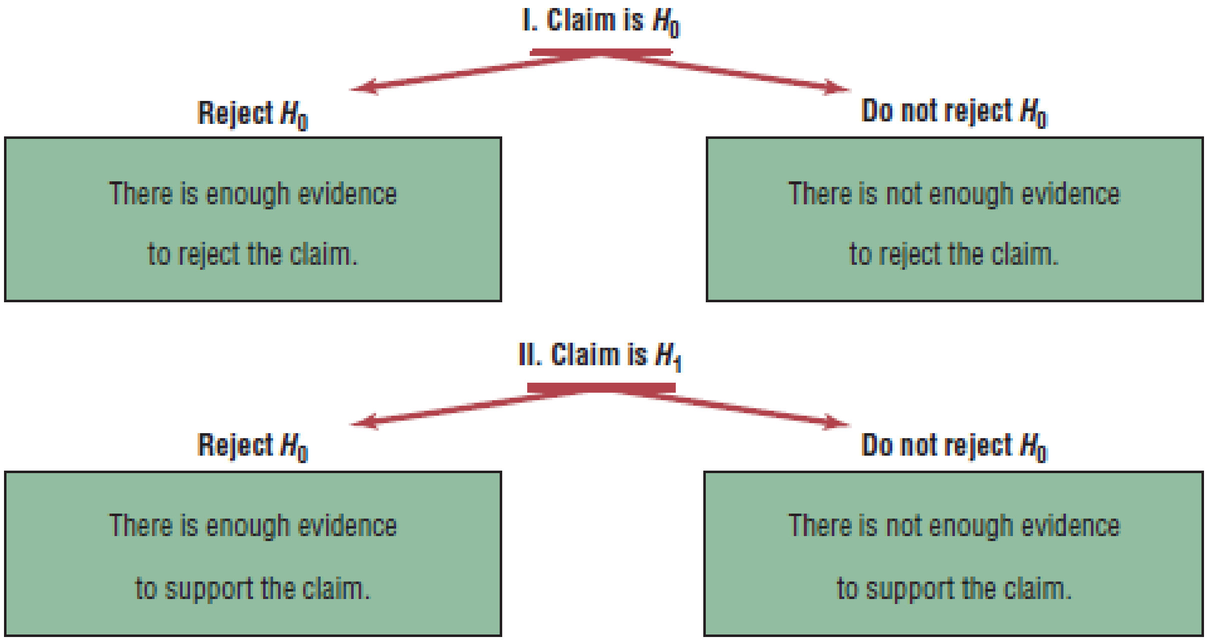


Step 5 Summarize the results. There is not enough evidence to support the claim that the mean time is greater than 29 days.

- Even though the sample mean of 30.1 is higher than the hypothesized population mean of 29, it is not significantly higher. Hence, the difference may be due to chance.
- When the null hypothesis is not rejected, there is still a probability of a **type II error**, i.e., of **not rejecting the null hypothesis when it is false**.

z Test For a Mean

- **z test**
- Also note that when the null hypothesis is not rejected, it cannot be accepted as true. **There is merely not enough evidence to say that it is false**.
- This guideline may sound a little confusing, but the situation is analogous to a jury trial. The verdict is either guilty or not guilty and is based on the evidence presented. If a person is judged not guilty, it does not mean that the person is proved innocent; it only means that there was not enough evidence to reach the guilty verdict.



z Test For a Mean

- **P-Value Method for Hypothesis Testing**
- **The P-value (or probability value)** is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.
- The P-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample statistic or a more extreme sample statistic occurring if the null hypothesis is true.
- **Example:** Suppose that an alternative hypothesis is $H_1: \mu > 50$ and the mean of a sample is $\bar{X} = 52$. If the computer printed a **P-value of 0.0356** for a statistical test, then the **probability of getting a sample mean of 52 or greater is 0.0356** if the true population mean is 50 (for the given sample size and standard deviation). The relationship between the P-value and the α value can be explained in this manner.
- For P=0.0356, the null hypothesis would be rejected at α=0.05 but not at α=0.01.

z Test For a Mean

- **P-Value Method for Hypothesis Testing**
- When the hypothesis test is two-tailed, the area in one tail must be doubled.
- For a two-tailed test, if α is 0.05 and the area in one tail is 0.0356, the P-value will be 2(0.0356)=0.0712. That is, the null hypothesis should not be rejected at α=0.05, since 0.0712 is greater than 0.05.
- In summary **if the P-value is less than α, reject the null hypothesis**. If the P-value is greater than a, do not reject the null hypothesis.
- The P-values for the z test can be found by using Standard Normal Distribution Tables.
- First find the area under the standard normal distribution curve corresponding to the z test value.
- For a left-tailed test, use the area given in the table; for a right-tailed test, use 1.0000 minus the area given in the table.
- To get the P-value for a two-tailed test, double the area you found in the tail.

z Test For a Mean

- **P-Value Method for Hypothesis Testing**
- **Solving Hypothesis-Testing Problems (P-Value Method):**
- Step 1 State the hypotheses and identify the claim.
- Step 2 Compute the test value.
- Step 3 Find the P-value.
- Step 4 Make the decision.
- Step 5 Summarize the results.
- **Decision Rule When Using a P-Value**

If P-value $\leq \alpha$, reject the null hypothesis.

If P-value $> \alpha$, do not reject the null hypothesis.

z Test For a Mean

- **P-Value Method for Hypothesis Testing**
- **Example:** A researcher wishes to test the claim that the average cost of tuition and fees at a four year public college is greater than \$5700. She selects a random sample of 36 four-year public colleges and finds the mean to be \$5950. The population standard deviation is \$659. Is there evidence to support the claim at $\alpha = 0.05$? Use the P-value method.
- **Solution:**

Step 1 State the hypotheses and identify the claim. H_0: $\mu = \$5700$ and H_1: $\mu > \$5700$ (claim).

Step 2 Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{5950 - 5700}{659/\sqrt{36}} = 2.28$$

Step 3 Find the *P*-value. Using Table E in Appendix C, find the corresponding area under the normal distribution for $z = 2.28$. It is 0.9887. Subtract this value for the area from 1.0000 to find the area in the right tail.

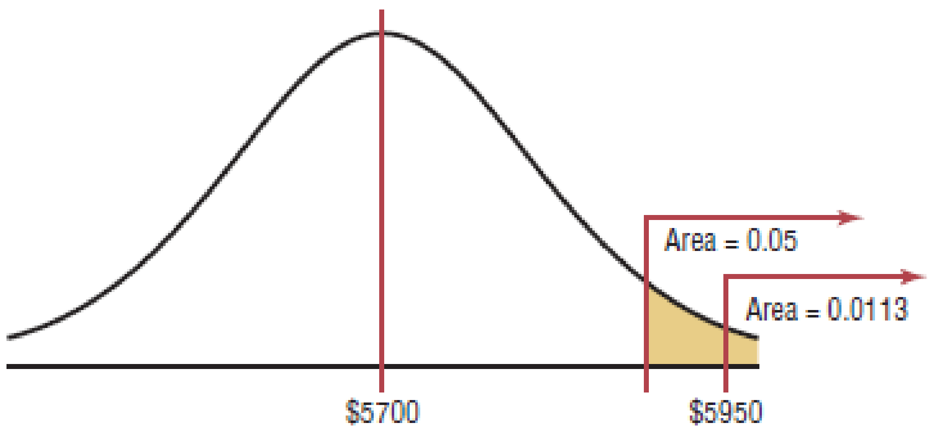
$1.0000 - 0.9887 = 0.0113$

Hence the *P*-value is 0.0113.

z Test For a Mean

- **P-Value Method for Hypothesis Testing**
- **Example:** A researcher wishes to test the claim that the average cost of tuition and fees at a four year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at $\alpha = 0.05$? Use the P-value method.
- **Solution:**

Step 4 Make the decision. Since the *P*-value is less than 0.05, the decision is to reject the null hypothesis.



Step 5 Summarize the results. There is enough evidence to support the claim that the tuition and fees at four-year public colleges are greater than $5700.

Note: Had the researcher chosen $\alpha = 0.01$, the null hypothesis would not have been rejected since the *P*-value (0.0113) is greater than 0.01.

t Test For a Mean

- **When the population standard deviation is unknown**, **t test** is used instead of the z test. The distribution of the variable should be approximately normal.

The ***t* test** is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, and σ is unknown.

The formula for the *t* test is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. $= n - 1$.

- The formula for the t test is similar to the formula for the z test. But since the population standard deviation **σ is unknown**, the **sample standard deviation s is used** instead.

t Test For a Mean

- For a one-tailed test, find the α level by looking at the top row of the table and finding the appropriate column. Find the degrees of freedom by looking down the left-hand column.
- Notice that the degrees of freedom are given for values from 1 through 30, then at intervals above 30. When the degrees of freedom are above 30, you should always round down to the nearest table value. For example, if d.f.=59, use d.f.=55 to find the critical value or values (**conservative approach**).

t Test For a Mean

- **Example:**

Find the critical t value for $\alpha = 0.05$ with d.f. $= 16$ for a right-tailed t test.

Solution

Find the 0.05 column in the top row and 16 in the left-hand column. Where the row and column meet, the appropriate critical value is found; it is $+1.746$.

d.f.	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1							
2							
3							
4							
5							
⋮							
14							
15							
16				1.746			
17							
18							
⋮							

t Test For a Mean

- **Example:**

Find the critical t value for $\alpha = 0.01$ with d.f. = 22 for a left-tailed test.

Solution

Find the 0.01 column in the row labeled One tail, and find 22 in the left column. The critical value is -2.508 since the test is a one-tailed left test.

- **Example:**

Find the critical values for $\alpha = 0.10$ with d.f. = 18 for a two-tailed t test.

Solution

Find the 0.10 column in the row labeled Two tails, and find 18 in the column labeled d.f. The critical values are $+1.734$ and -1.734.

- **Example:**

Find the critical value for $\alpha = 0.05$ with d.f. = 28 for a right-tailed t test.

Solution

Find the 0.05 column in the One-tail row and 28 in the left column. The critical value is $+1.701$.

t Test For a Mean

- **Example:** A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at α=0.05?

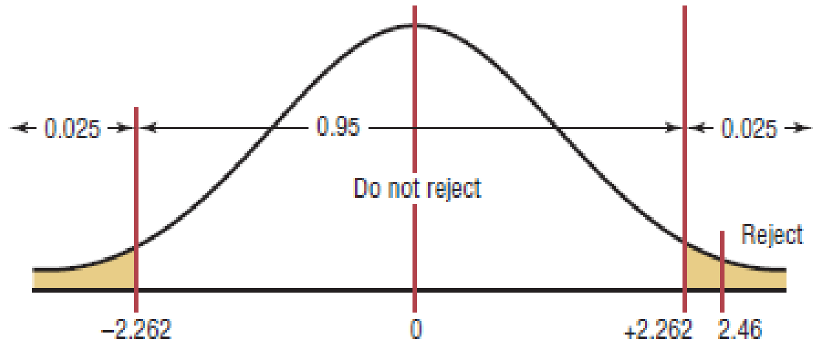
Step 1 H_0: $\mu = 16.3$ (claim) and H_1: $\mu \neq 16.3$.

Step 2 The critical values are +2.262 and −2.262 for $\alpha = 0.05$ and d.f. = 9.

Step 3 The test value is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{17.7 - 16.3}{1.8/\sqrt{10}} = 2.46$$

Step 4 Reject the null hypothesis since 2.46 > 2.262. See Figure 8–22.



Step 5 There is enough evidence to reject the claim that the average number of infections is 16.3.

t Test For a Mean

- The P-values for the t test can be found by using **t distribution**.
- However, specific P-values for t tests cannot be obtained from the table since only selected values of α.
- **Example:** Find the P-value when the t test value is 2.056, the sample size is 11, and the test is right-tailed.
- **Solution:**
- To get the P-value, look across the row with 10 degrees of freedom (d.f.=n-1) in t distribution and find the two values that 2.056 falls between. They are 1.812 and 2.228.
- Since this is a right-tailed test, look up to the row labeled One tail, a and find the two a values corresponding to 1.812 and 2.228. They are 0.05 and 0.025, respectively.

t Test For a Mean

- **Example:** Find the P-value when the t test value is 2.056, the sample size is 11, and the test is right-tailed.
- **Solution:**

d.f.	Confidence intervals	50%	80%	90%	95%	98%	99%
	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		0.816	1.886	2.920	4.303	6.965	9.925
3		0.765	1.638	2.353	3.182	4.541	5.841
4		0.741	1.533	2.132	2.776	3.747	4.604
5		0.727	1.476	2.015	2.571	3.365	4.032
6		0.718	1.440	1.943	2.447	3.143	3.707
7		0.711	1.415	1.895	2.365	2.998	3.499
8		0.706	1.397	1.860	2.306	2.896	3.355
9		0.703	1.383	1.833	2.262	2.821	3.250
10		0.700	1.372	1.812 *	2.228	2.764	3.169
11		0.697	1.363	1.796	2.201	2.718	3.106
12		0.695	1.356	1.782	2.179	2.681	3.055
13		0.694	1.350	1.771	2.160	2.650	3.012
14		0.692	1.345	1.761	2.145	2.624	2.977
15		0.691	1.341	1.753	2.131	2.602	2.947
⋮		⋮	⋮	⋮	⋮	⋮	⋮
(z) ∞		0.674	1.282	1.645	1.960	2.326	2.576

*2.056 falls between 1.812 and 2.228.

t Test For a Mean

- **Example:** Find the P-value when the t test value is 2.056, the sample size is 11, and the test is right-tailed.
- **Solution:**
- Hence, the P-value would be contained in the interval $0.025 < P\text{-value} < 0.05$. This means that the P-value is between 0.025 and 0.05. If a were 0.05, you would reject the null hypothesis since the P-value is less than 0.05. But if α were 0.01, you would not reject the null hypothesis since the P-value is greater than 0.01. (Actually, it is greater than 0.025.)
- **Example:** Find the P-value when the t test value is 2.983, the sample size is 6, and the test is two-tailed.

Solution

To get the *P*-value, look across the row with d.f. = 5 and find the two values that 2.983 falls between. They are 2.571 and 3.365. Then look up to the row labeled Two tails, α to find the corresponding α values.

In this case, they are 0.05 and 0.02. Hence the *P*-value is contained in the interval $0.02 < P\text{-value} < 0.05$. This means that the *P*-value is between 0.02 and 0.05. In this case, if $\alpha = 0.05$, the null hypothesis can be rejected since $P\text{-value} < 0.05$; but if $\alpha = 0.01$, the null hypothesis cannot be rejected since $P\text{-value} > 0.01$ (actually $P\text{-value} > 0.02$).

t Test For a Mean

- **Example:** A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at α=0.05?
- **Solution:**

Step 1 State the hypotheses and identify the claim.

H_0: $\mu = 36.7$ and H_1: $\mu > 36.7$ (claim)

Step 2 Compute the test value. The test value is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{40.6 - 36.7}{6/\sqrt{15}} = 2.517$$

Step 3 Find the P-value. Looking across the row with d.f. = 14 in Table F, you see that 2.517 falls between 2.145 and 2.624, corresponding to $\alpha = 0.025$ and $\alpha = 0.01$ since this is a right-tailed test. Hence, P-value > 0.01 and P-value < 0.025, or $0.01 < P\text{-value} < 0.025$. That is, the P-value is somewhere between 0.01 and 0.025. (The P-value obtained from a calculator is 0.012.)

Step 4 Reject the null hypothesis since P-value < 0.05 (that is, P-value < α).

Step 5 There is enough evidence to support the claim that the joggers' maximal volume oxygen uptake is greater than 36.7 ml/kg.

t Test For a Proportion

- A hypothesis test involving a population proportion can be considered as a binomial experiment when there are only two outcomes and the probability of a success does not change from trial to trial.
- Since a normal distribution can be used to approximate the binomial distribution when $np \geq 5$ and $nq \geq 5$, the standard normal distribution can be used to test hypotheses for proportions.
- **Formula for the z Test for Proportions**
-

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

where

$$\hat{p} = \frac{X}{n} \qquad \text{(sample proportion)}$$

p = population proportion
n = sample size

t Test For a Proportion

- **Example:** A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At α=0.05, is there enough evidence to reject the dietitian's claim?
- **Solution:**
- **Step 1** State the hypothesis and identify the claim.

  H_0: $p = 0.60$ (claim) and H_1: $p \neq 0.60$

  Step 2 Find the critical values. Since $\alpha = 0.05$ and the test value is two-tailed, the critical values are ± 1.96.

  Step 3 Compute the test value. First, it is necessary to find $\hat{p}$.

$$\hat{p} = \frac{X}{n} = \frac{128}{200} = 0.64 \qquad p = 0.60 \qquad q = 1 - 0.60 = 0.40$$

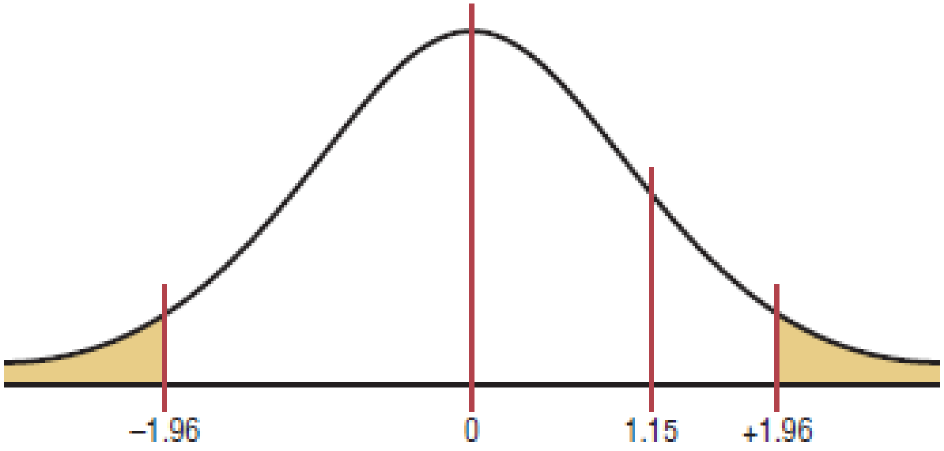
  Substitute in the formula.

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.64 - 0.60}{\sqrt{(0.60)(0.40)/200}} = 1.15$$

  Step 4 Make the decision. Do not reject the null hypothesis since the test value falls outside the critical region

t Test For a Proportion

- **Example:** A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At α=0.05, is there enough evidence to reject the dietitian's claim?
- **Solution:**
-



Step 5 Summarize the results. There is not enough evidence to reject the claim that 60% of people are trying to avoid trans fats in their diets.

χ^2 Test For a Variance or Standard Deviation

- A chi-square distribution is used to test a claim about a single variance or standard deviation.
- **Formula for the Chi-Square Test for a Single Variance**
- $$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

  with degrees of freedom equal to $n - 1$ and where

  n = sample size

  s^2 = sample variance

  σ^2 = population variance

- **Assumptions for the Chi-Square Test for a Single Variance**
- The sample must be randomly selected from the population.
- The population must be normally distributed for the variable under study.
- The observations must be independent of one another.

χ^2 Test For a Variance or Standard Deviation

- There are several reasons to test variances.
- First, in any situation where consistency is required, such as in manufacturing, you would like to have the smallest variation possible in the products.
- For example, when bolts are manufactured, the variation in diameters due to the process must be kept to a minimum, or the nuts will not fit them properly. In education, consistency is required on a test. That is, if the same students take the same test several times, they should get approximately the same grades, and the variance of each of the student's grades should be small.
- On the other hand, if the test is to be used to judge learning, the overall standard deviation of all the grades should be large so that you can differentiate those who have learned the subject from those who have not learned it.

χ^2 Test For a Variance or Standard Deviation

- **Example:** An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance ($\sigma^2 = 2.25$) at α=0.05? Assume that the scores are normally distributed.

Solution

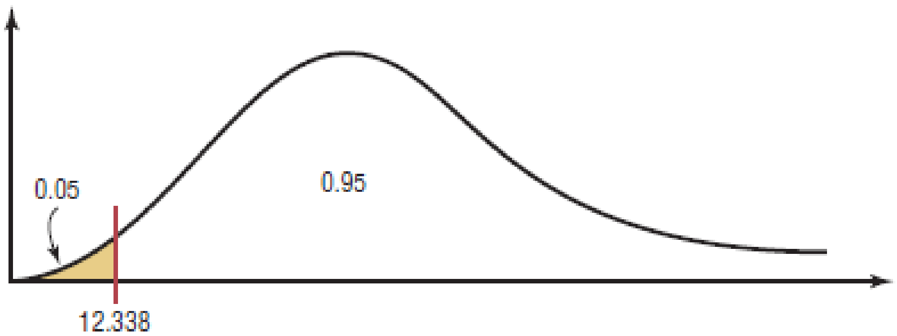
Step 1 State the hypotheses and identify the claim.

H_0: $\sigma^2 = 225$ and H_1: $\sigma^2 < 225$ (claim)

Step 2 Find the critical value. Since this test is left-tailed and $\alpha = 0.05$, use the value $1 - 0.05 = 0.95$. The degrees of freedom are $n - 1 = 23 - 1 = 22$. Hence, the critical value is 12.338. Note that the critical region is on the left

χ^2 Test For a Variance or Standard Deviation

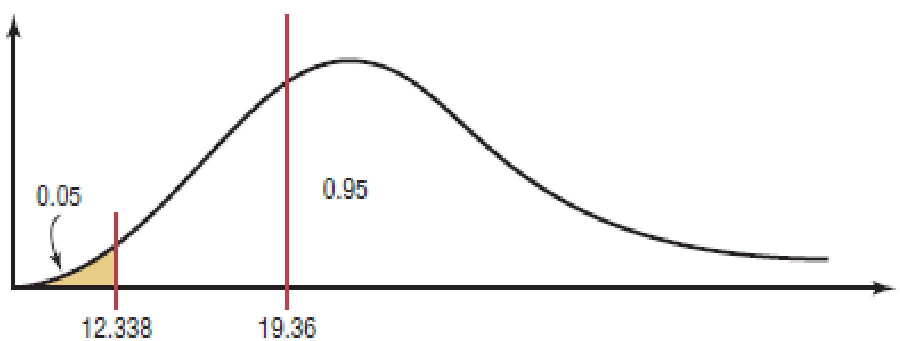
- **Solution:**



Step 3 Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(23-1)(198)}{225} = 19.36$$

Step 4 Make the decision. Since the test value 19.36 falls in the noncritical region, as shown in Figure 8–36, the decision is to not reject the null hypothesis.



Step 5 Summarize the results. There is not enough evidence to support the claim that the variation in test scores of the instructor's students is less than the variation in scores of the population.

χ^2 Test For a Variance or Standard Deviation

- Approximate P-values for the chi-square test can be found by using chi-square distribution.
- The procedure is somewhat more complicated than the previous procedures for finding P-values for the z and t tests since the chi-square distribution is not exactly symmetric and χ^2 values cannot be negative.
- As we did for the t test, we will determine an interval for the P-value based on the table.
- When the χ^2 test is two-tailed, both interval values must be doubled.
- **Example:**

Find the *P*-value when $\chi^2 = 19.274$, $n = 8$, and the test is right-tailed.

Solution

To get the *P*-value, look across the row with d.f. = 7 in Table G and find the two values that 19.274 falls between. They are 18.475 and 20.278. Look up to the top row and find the α values corresponding to 18.475 and 20.278. They are 0.01 and 0.005, respectively. See Figure 8–40. Hence the *P*-value is contained in the interval $0.005 < P\text{-value} < 0.01$. (The *P*-value obtained from a calculator is 0.007.)

χ^2 Test For a Variance or Standard Deviation

- **Example:**

Degrees of freedom	α									
	0.995	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

*19.274 falls between 18.475 and 20.278

χ^2 Test For a Variance or Standard Deviation

- **Example:** A researcher knows from past studies that the standard deviation of the time it takes to inspect a car is 16.8 minutes. A sample of 24 cars is selected and inspected. The standard deviation is 12.5 minutes. At α=0.05, can it be concluded that the standard deviation has changed? Use the P-value method.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: $\sigma = 16.8$ and H_1: $\sigma \neq 16.8$ (claim)

Step 2 Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24-1)(12.5)^2}{(16.8)^2} = 12.733$$

Step 3 Find the *P*-value. Using Table G with d.f. = 23, the value 12.733 falls between 11.689 and 13.091, corresponding to 0.975 and 0.95, respectively. Since these values are found on the left side of the distribution, each value must be subtracted from 1. Hence $1 - 0.975 = 0.025$ and $1 - 0.95 = 0.05$. Since this is a two-tailed test, the area must be doubled to obtain the *P*-value interval. Hence $0.05 < P\text{-value} < 0.10$, or somewhere between 0.05 and 0.10. (The *P*-value obtained from a calculator is 0.085.)

χ^2 Test For a Variance or Standard Deviation

- **Solution:**

Step 4 Make the decision. Since $\alpha = 0.05$ and the P-value is between 0.05 and 0.10, the decision is to not reject the null hypothesis since P-value $> \alpha$.

Step 5 Summarize the results. There is not enough evidence to support the claim that the standard deviation has changed.